

A Grid-Based Fitness Strategy for Evolutionary Many-Objective Optimization

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ABSTRACT

Grid has been widely used in the field of evolutionary multi-objective optimization (EMO) due to its property combining convergence and diversity naturally. Most EMO algorithms of grid-based fitness perform well on problems with two or three objectives, but encounter difficulties in their scalability to many-objective optimization. This paper develops the potential of using grid technique to balance convergence and diversity in fitness for many-objective optimization problems. To strengthen selection pressure and refine comparison level, three hierarchical grid-based criterions are incorporated into fitness to establish a complete order among individuals. Moreover, an adaptive fitness penalty mechanism in environmental selection is employed to guarantee the diversity of archive memory. Based on an extensive comparative study with three other EMO algorithms, the proposed algorithm is found to be remarkably successful in finding well-converged and well-distributed solution set.

Categories and Subject Descriptors

I.2.8 [Computing Methodologies]: Problem Solving, Control Methods, and Search

General Terms

Algorithms, Performance, Experimentation.

Keywords: Multiobjective optimization, many-objective optimization, fitness assignment, grid

1. INTRODUCTION

Fitness assignment is one of the most important components in evolutionary multiobjective optimization algorithms (EMOAs). Over the past few years, numerous fitness assignment techniques have been proposed and performed successfully in the two

fundamental purposes of evolutionary multiobjective optimization (EMO): (i) convergence towards the optimal front and (ii) maintenance of a set of well-distributed solutions.

Most EMOAs calculate fitness value mainly on the basis of the Pareto dominance relation, i.e., the information which individuals dominates, is dominated or nondominated is used to define a rank [3, 6]. Additionally, diversity information that is closely related to the density estimator of the individuals is usually also recognized as an auxiliary consideration to be incorporated into the fitness rank. This general template of fitness assignment has already been proven to be very efficient for two- and three-dimensional objective problems.

Unfortunately, when the problems have more than three objectives, which are also termed *many-objective* problems [11], the EMOAs with the above template of fitness assignment have difficulties and challenges to find a good approximation of Pareto front [16], such as NSGA-II [8] and SPEA2 [26]. Some recent studies have pointed out these EMOAs may even perform worse than the random search optimizer for the problems with 10 or more objectives [4, 18, 19]. One of the main reasons for this occurrence is that the proportion of nondominated solutions in a population rises rapidly with the increasing of the number of objectives [6, 14]. Thus, the primary consideration of fitness assignment (i.e. Pareto dominance relation) would fail to distinguish these nondominated solutions and cannot select the proper candidate solutions for searching towards the Pareto front. In this case, diversity information would become the primary selection criterion and play a leading role in determining the survival of individuals [1, 23]. However, most of existing diversity maintenance techniques, such as Niche, Crowding Distance, clustering, the k -th nearest neighbor, and so forth [3], not only cannot increase the selection pressure towards the Pareto front, but hinder evolutionary search to some extent due to their overemphasis of extensive distribution [13]. Even extreme solutions are often preferable in these techniques since one or more objectives of them are significantly worse than that of the other solutions and hence located in sparse areas. In fact, Purchase and Fleming have shown that the selection mechanisms without density estimator may achieve better convergence results in many-objective problems [21].

A straightforward idea to overcome this difficulty is to adopt some other optimality relations replacing or enhancing Pareto dominance

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relation in fitness assignment process in order to increase the selection pressure towards the Pareto front; such as, average ranking [2], k-optimality [12], preference order ranking [20], favour relation [10], and some methods that control the dominance area [22]. However, as lack of effective diversity maintenance mechanism, the optimal set obtained by these relations is usually a subset of the Pareto optimal set [15]. Moreover, although some modifications of diversity strategies could improve the convergence by way of punishing the extreme solutions [23] or adjusting diversity promotion operator [1], these techniques cannot get to the root of the problem. In addition, some Hypervolume-based EMOAs have recently been reported good results for many-objective problems [23]. However, the large computation cost of the hypervolume calculation may limit their applicability. Anyway, as indicated by Hughes [13], to many-objective problems it is not a trivial job to assign a fitness value that can provide sufficient selective pressure towards the Pareto front while also supply an effective drive towards a set of well-spread solutions.

Grid has been widely used in the field of EMO due to its property combining convergence and diversity naturally. Existing grid-based fitness strategies have been proven to perform well on problems with two or three objectives. In this paper, we expand its potential to many-objective problems. In order to balance convergence and diversity in high dimensional space, we present a fitness assignment strategy that defines three grid-based relations: *grid ranking*, *grid crowding degree* and *grid coordinate point distance* to compare and distinguish individuals. Moreover, we also propose a fitness adjustment technique to ensure the diversity of offspring population by punishing the individuals once their neighbors have been picked out into the archive. In section 2, the motivation of this strategy is delineated. Section 3 is devoted to the description of our proposal. Experimental results are presented and analyzed in section 4. Finally, section 5 concludes the paper and gives some lines of future work.

2. MOTIVATION

Although the difficulties of fitness assignment in many-objective problems are manifest, grid seems to be a natural way to combine convergence and diversity in evolutionary search since each individual has a determinate location (i.e., coordinates) in it¹. On one hand, the coordinate not only reflects the number of objectives for which one individual is better than the other, but implies the difference in objective values between the two individuals. For instance, for the individuals **A** and **B** in Figure 1, the grid coordinates of them are (0, 4) and (3, 3), respectively. Thus, it is clear that the difference of the two individuals in objective f_1 should be greater than that in objective f_2 . However, Pareto dominance relation that only considers the pros and cons fails to reflect this difference. On the other hand, grid is also capable of indicating the distribution information of individuals in it. Considering the hyperbox with the grid coordinate (3, 1) in Figure 1, obviously, it is more crowded than other ones in grid because it contains the largest number of individuals (**C**, **D** and **E**).

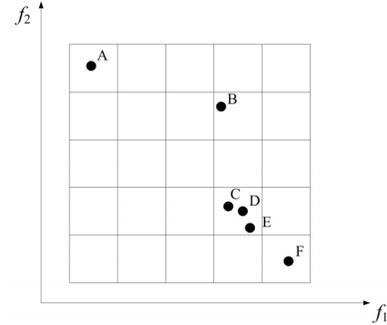


Figure 1: An example of distribution of individuals in grid

However, it is surprising that the application of grid-based technique to many-objective problems has received little attention and consideration, and the existing grid-based EMOAs such as PESA [5] and AGA [17] only work well on the problems with two or three objectives. This occurrence may be mainly attributed the reason that, as pointed out by Corne and Knowles [4], for most grid-based EMOAs, their operation relies on data-structures that grow exponentially in the number of objectives. The computational cost for high dimensional problems would be tremendous when grid-centered calculation is implemented [16]. If we traverse each hyperbox in k -dimension grid, there will be r^k hyperboxes to be accessed, where r is the divisions in each dimension. Thus, a small number of divisions are forced to use in many-objective problems. Unfortunately, however, since the fitness information of these algorithms depends seriously on the number of individuals occupying a hyperbox (i.e., the individuals in less crowded hyperboxes would be preferred), the small number of division will result in reducing the ability of selection to provide effective discrimination [4].

However, we argue that the difficulties of grid-based techniques for many-objective problems do not seem to be insurmountable, if an individual-centered calculation is adopted to take the place of grid-centered calculation. In this case, grid is merely regarded as an implement to depict the address of individuals. Moreover, the selection pressure may also be increased, if the fitness values of individuals do not rely on the records in single hyperbox but the relative position in whole evolutionary population.

Bearing these ideas and motivations in mind, a novel grid-based fitness assignment and adjustment technique for many-objective problems is suggested and described in the next section.

3. THE PROPOSED METHOD

In this study, grid is envisaged as an implement or a frame more specifically to determine the location of individuals in the objective space. Thus, the adaptability of it varying with the evolution of individuals seems to be more advisable. In other words, as individuals in the objective space are generated, the location and size of a grid should be adapted and adjustable so that it just envelops the individuals. Here, we adopt the adaptive construction of grid borrowing from the AGA algorithm presented by Knowles and Corne [17].

¹ In this paper, the grid coordinate of individual is denoted by that of the hyperbox which contains it.

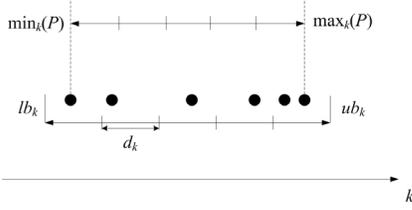


Figure 2: The setting of grid in the k th objective

The setting of grid in the k th objective is shown in Figure 2. First the minimum and maximum values of the objective k amongst the individuals in a population set P are found and denoted as $\min_k(P)$ and $\max_k(P)$, respectively. Afterward, the lower and upper boundaries in the k th objective are determined by them:

$$lb_k = \min_k(P) - (\max_k(P) - \min_k(P)) / (2 \times div) \quad (1)$$

$$ub_k = \max_k(P) + (\max_k(P) - \min_k(P)) / (2 \times div) \quad (2)$$

where div is a constant parameter, the number of divisions of the objective space in each dimension, set by the user (e.g., in Figure 2, $div = 5$). Accordingly, the original M -dimensional objective space will be divided into div^M hyperboxes. Thus, the hyperbox width in the k th objective, d_k , can be formed as

$$d_k = (ub_k - lb_k) / div \quad (3)$$

Therefore, according to lb_k and d_k , the grid coordinate of any individual in the k th objective is determined as

$$G_k(\mathbf{A}) = \lfloor (F_k(\mathbf{A}) - lb_k) / d_k \rfloor \quad (4)$$

where $G_k(\mathbf{A})$ is the grid coordinate of individual \mathbf{A} in the k th objective, $F_k(\mathbf{A})$ is the actual objective value in the k th objective. For instance, in Figure 2, the grid coordinates of all individuals (from left to right) in the k th objective are 0, 1, 2, 3, 4 and 4, respectively. A more detailed description about the setting of adaptive grid can be referred in the AGA algorithm [17].

3.1 FITNESS ASSIGNMENT

After the address of each individual in the population has been determined, the fitness of each individual would be calculated according to it. In this paper, we take into account three grid-based relations to evaluate and compare individuals. They are grid ranking (GR), grid crowding degree (GCD) and grid coordinate point distance (GCPD). In the following, we will introduce them orderly.

GR is a simple convergence estimator to distinguish and rank individuals in the light of their address. It is defined as the summation of the grid coordinates of individuals:

$$GR(\mathbf{A}) = \sum_{k=1}^M G_k(\mathbf{A}) \quad (5)$$

where $G_k(\mathbf{A})$ denotes the grid coordinate of individual \mathbf{A} in the k th objective, and M is the number of objectives. Clearly, a lower GR of individual is preferable. As the grid coordinate of individual in each objective is determined actually by the evolutionary population and the numbers of divisions in different objectives are also equal, similar to the normalization process, the comparison of grid coordinates of individuals in different objectives is feasible and reasonable. Therefore, the summation of the grid coordinates of individual would reflect its evolution degree compared with other

individuals in the population, and be influenced by two factors specifically: the number of objectives and the level in single objective. In one respect, GR prefers the individuals with good performance in more objectives. An individual with better performance in more objectives would have a higher likelihood to achieve a lower GR value. On the other hand, the difference in single objective among individuals is also an important part of determining the GR value. For instance, in Figure 3, considering the nondominated individuals \mathbf{D} and \mathbf{A} , as the advantage in f_2 is less than disadvantage in f_1 , \mathbf{D} will obtain a worse GR value than \mathbf{A} (5 against 4). In conclusion, GR can be regarded as a natural tradeoff by integrating the number of objectives for which one solution is better than the other and the difference in objective values between them, and hence significantly enhances the selection pressure towards the optimization direction.

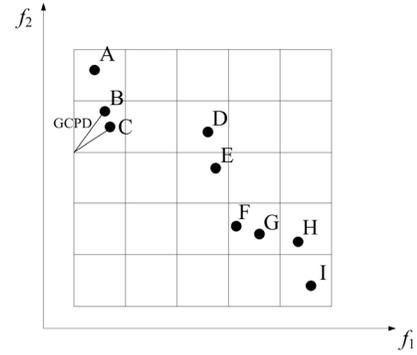


Figure 3: Illustration of fitness assignment. The GR and GCD for these individuals are A (4, 2), B(3, 2), C(3, 2), D(5, 1), E(4, 1), F(4, 2), G(4, 2), H(5, 3), and I(4, 1), the left value in the brackets associated with GR.

As the number of hyperboxes grows exponentially with the number of objectives, the density estimation of the existing grid-based EMOAs, which records the number of individuals occupying a hyperbox may fail to reveal the distribution of them, because the individuals have a very high likelihood of locating in distinct hyperboxes. Here, we enlarge the recording region to discriminate individuals effectively. A group of neighboring hyperboxes of individual is taken as a niche to be considered (called this the *grid niche*). The individuals in the grid niche of an individual are regarded as its neighbors, and further the grid crowding degree (GCD) of it is defined by the number of its neighbors²:

$$GCD(\mathbf{X}) = \left| \left\{ \mathbf{Y} \mid \mathbf{Y} \neq \mathbf{X}, \sum_{k=1}^M |G_k(\mathbf{X}) - G_k(\mathbf{Y})| < M \right\} \right| \quad (6)$$

where $|\cdot|$ denotes the cardinality of a set, $G_k(\mathbf{X})$ implies the grid coordinates of individual \mathbf{X} in the k th objective, M is the number of objectives, and \mathbf{Y} corresponds to the neighbors of \mathbf{X} . For instance, in Figure 3, the neighbor of individual \mathbf{D} is \mathbf{E} and the GCD value of \mathbf{D} is 1; the neighbors of individual \mathbf{G} are \mathbf{F} and \mathbf{H} , and hence the GCD value of \mathbf{G} is 2. Correspondingly, the grid niche of \mathbf{D} is composed by hyperboxes (2, 3), (1, 3), (3, 3), (2, 2) and (2, 4), and the grid

² As the grid coordinate of individual is denoted by that of the hyperbox which contains it, the definition of grid niche of individual can also be obtained from (6), and hence omitted for brevity.

niche of \mathbf{G} is composed by hyperboxes (3, 1), (3, 0), (3, 2), (2, 1) and (4, 1). Note that the range of grid niche of individuals is determined by variable M . The number of hyperboxes in grid niche of individual will gradually increase with the number of objectives, which is consistent with the total number of hyperboxes in grid environment, and hence providing a finer distinction about the degree of crowding among individuals.

In addition, it is necessary to mention here that GCD is merely regarded as an auxiliary criterion to compare individuals. In other words, only when the primary criterion GR of individuals is uncomparable (i.e., equal), the GCD is considered to distinguish them. For instance, the individuals \mathbf{D} and \mathbf{H} have equal GR value (5) in Figure 3. The difference of GCD (1 against 3) indicates that individual \mathbf{D} is preferable to \mathbf{H} .

Although GR and GCD have already provided a rough measure for individuals in terms of convergence and diversity, they may also fail to discriminate the individuals, e.g., the individuals located in identical hyperbox. Here, borrowing from ε -MOEA [7], we calculate the distance between the individual and the point of its grid coordinates to refine convergence estimation. Specifically, it namely grid coordinate point distance (GCPD), is defined as follows:

$$GCPD(\mathbf{A}) = \sqrt{\sum_{k=1}^M (F_k(\mathbf{A}) - (lb_k + G_k(\mathbf{A}) \times d_k))^2} \quad (7)$$

where $G_k(\mathbf{A})$ denotes the grid coordinate of individual \mathbf{A} in the k th objective, $F_k(\mathbf{A})$ stands for the actual objective value in the k th objective, lb_k implies the lower boundary in the k th objective, d_k corresponds to the hyperbox width in the k th objective, and M is the number of objectives. Individuals \mathbf{B} and \mathbf{C} in Figure 3 illustrate this case. Clearly, a shorter GCPD of individuals is preferable when the other two criterions are uncomparable.

In conclusion, according to the above three different levels of comparison criterions within fitness, it would be able to establish a complete order among individuals for further selection operation.

3.2 FITNESS ADJUSTMENT IN ENVIRONMENTAL SELECTION

Unlike mating selection in evolutionary process, which aims at picking promising individuals for variation and usually is performed in a randomized fashion, environmental selection directly determines which of the previously stored individuals and the newly created ones kept in the archive memory [25]. Therefore, the straightforward manner according to fitness level may lead individuals to concentrate in a subregion of the current front in the archive memory. Here, we present a fitness adjustment strategy by adaptively punishing the individuals once their neighbors have been picked out into the archive. More precise, assuming individual \mathbf{Y} is a neighbor of individual \mathbf{X} , when \mathbf{Y} has entered the archive, the adjustment of GR of \mathbf{X} is implemented as follows:

$$GR'(\mathbf{X}) = GR(\mathbf{X}) + (M - \sum_{k=1}^M |G_k(\mathbf{X}) - G_k(\mathbf{Y})|) \quad (8)$$

where $G_k(\mathbf{X})$ denotes the grid coordinate of individual \mathbf{X} in the k th objective, and M is the number of objectives. Note that only the GR of individuals varies in adjustment process, while the other fitness

information (i.e., GCD and GCPD) of individuals stays the same. In order to clearly understand this adjustment scheme, an illustration of environmental selection process for the case in Figure 3 is shown in Figure 4. First, \mathbf{C} is picked out into the archive since it has the best fitness value ($GR(\mathbf{C}) = GR(\mathbf{B}) = 3$, $GCD(\mathbf{C}) = GCD(\mathbf{B}) = 2$, $GCPD(\mathbf{C}) < GCPD(\mathbf{B})$). Correspondingly, the GR of the neighbors \mathbf{A} , \mathbf{B} of \mathbf{C} is penalized from (8) shown in Figure 4(b). Again, \mathbf{I} is picked out since it performs best in current candidate individuals ($GR(\mathbf{I}) = GR(\mathbf{E}) = GR(\mathbf{F}) = GR(\mathbf{G}) = 4$, $GCD(\mathbf{I}) = GCD(\mathbf{E}) = 1 < GCD(\mathbf{F}) = GCD(\mathbf{G}) = 2$, $GCPD(\mathbf{I}) < GCPD(\mathbf{E})$). Then the GR of the neighbor \mathbf{H} of \mathbf{I} is adjusted similarly shown in Figure 4(c). Repeat this procedure until the predefined size is achieved. The final individuals in the archive are \mathbf{A} , \mathbf{C} , \mathbf{E} , \mathbf{F} and \mathbf{I} . Clearly, by continuous selection and fitness adjustment, a tradeoff between convergence and diversity would be obtained. More specifically, from (8), we can draw two in-depth characteristics of the fitness adjustment scheme as follows.

- The punishment of individual is related to the differences of grid coordinates between it and the selected neighbor. The neighbors located in same hyperbox with the selected individual are subjected to the most severe penalty, and the farther the neighbors located, the milder is the punishment.
- Since the adjustment is also connected with variable M , the overall level of punishment in grid niche will be aggravated with the number of objectives. This behavior seems to be reasonable because the differences of GR among individuals perform more obviously in the problems with higher dimensions. An increasing penalty could develop wider selection among the candidate individuals and prevent them from converging to a few regions in the archive.

3.3 TIME REQUIREMENTS OF THE PROPOSED METHOD

The computational cost of the proposed method can be divided into three parts: grid construction, fitness assignment and environmental selection. The grid construction procedure demands to identify the maximum and minimum value in each objective for the population of size N . This requires $O(MN)$ comparison where M is the number of objectives. The computational cost of fitness assignment is determined by three comparison criterions: GR, GCD and GCPD. Clearly, from (5) and (7), the time complexities of calculating GR and GCPD for all individuals are both $O(MN)$. As the calculation of GCD for each individual demands to traverse population from (6), the time complexity of it for all individuals is $O(MN^2)$, which will govern the computational cost of fitness assignment. The environmental selection procedure demands to adjust fitness value as well as find the next best individual when the current best individual has entered the archive. This requires $O(MN)$ computations. Thus, the time complexity for filling archive is $O(MN^2)$, assuming the archive size is also equal to N .

From the analysis above, the total complexity of the proposed method is $O(MN^2)$. This cost is fully determined by the population size and the number of objectives, thereby being independent of the divisions of grid and not being added with the number of hyperboxes.

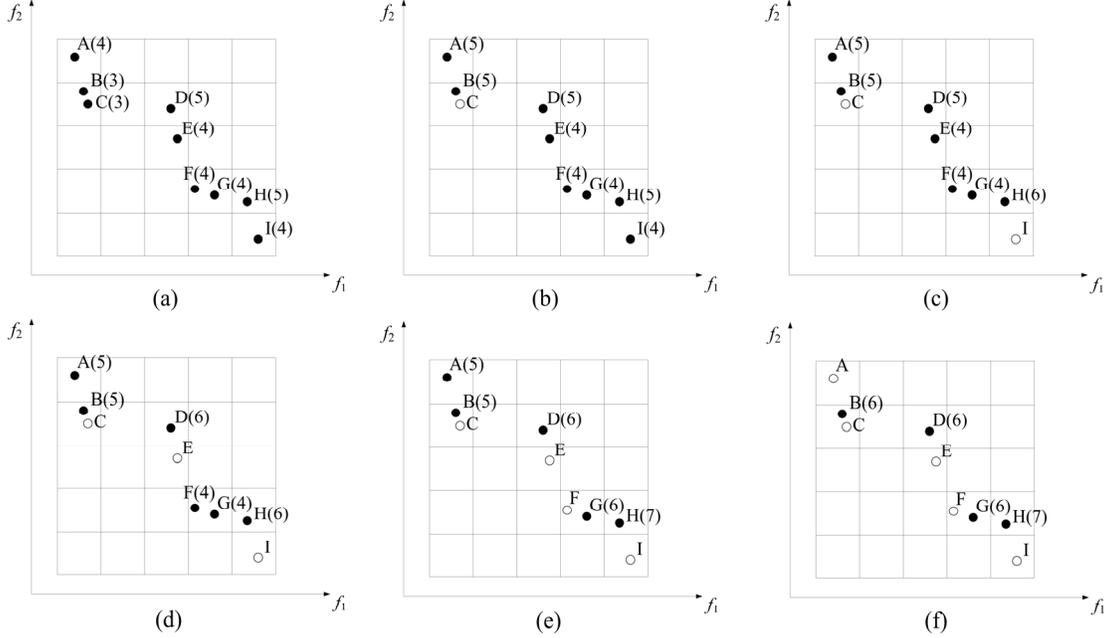


Figure 4: An illustration of fitness adjustment in environmental selection process. Where archive size is set to 5. The value in the brackets corresponds to GR of individuals. Black points stand for the candidate individuals for archive and hollow points stand for the individuals that have entered the archive set.

4. COMPARATIVE STUDY

In order to validate the proposed method, we compare it with other three algorithms for solving many-objective problems: NSGA-II [8], average ranking (AR) [2] and average ranking with improved crowding distance (AR+CD') [23].

NSGA-II is one of the most popular EMOAs. The characteristic feature of it is fast nondominated sorting and crowding distance density estimator in fitness assignment for mating selection and environmental selection. The AR strategy compares all individuals on each objective and ranks them independently. The final rank of an individual is obtained by summing all its ranks on each objective. Corne and Knowles have reported that this strategy performs better than some more complicated ranking ones [4]. Here, we introduce two AR-based algorithms for comparison. One, similar to the design in [4], employs AR to select individuals for variation and renews the archive in random way. The other algorithm is also AR-based comparison in mating selection, yet adopts an improved crowding distance for environmental selection, which is the assignment of a zero distance (instead of an infinity distance) to extreme solutions in order to advance the convergence of algorithm [23].

For a fair comparison, the proposed method, AR and AR+CD' are embedded into NSGA-II template. Parent population (which can actually be regarded as the archive) combines with current population for generating the best 50% offspring. The last allowed nondominated front is considered by the above strategies instead of crowding distance in environmental selection. Additionally, binary tournament selection based on their fitness information is used to fill the mating pool. In the following, two performance metrics and test problem used in comparison are introduced in brief.

4.1 PERFORMANCE METRICS AND TEST PROBLEM

In this paper, convergence metric proposed by Deb et al. [7] and diversity metric proposed by Adra et al. [1] are considered. The convergence metric calculates the average distance of the obtained solutions set away from the Pareto front. Similar to the studies in [16, 23], the distance to the Pareto front is determined analytically without using a reference set.

The diversity metric is an improved *Maximum Spread* (MS) evaluation considering the distribution of the Pareto front [1]. The original MS [24] measures the length of the diagonal of the hypercube formed by the extreme objective values in a given set. The improved MS (MS') calculates the ratio of MS between the obtained solution set and the Pareto front:

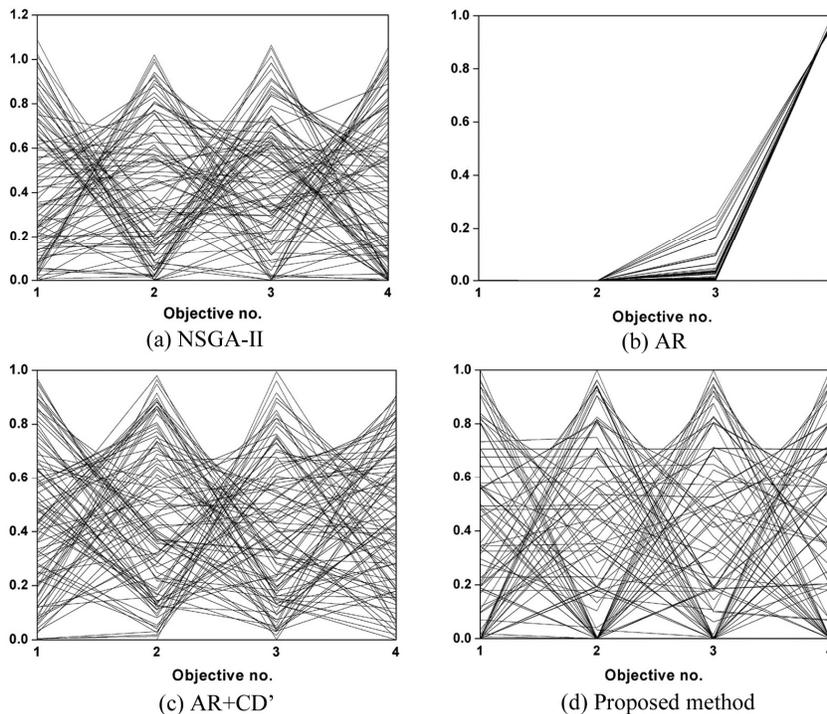
$$MS' = \sqrt{\sum_{m=1}^M (\max_{p \in P} (p_m) - \min_{p \in P} (p_m))^2} / \sqrt{\sum_{m=1}^M (\max_{q \in P^*} (q_m) - \min_{q \in P^*} (q_m))^2} \quad (9)$$

where M is the number of objectives, P denotes the obtained solution set and P^* stands for the Pareto front. Clearly, an indicator value close to one ($MS' = 1$) is desired. Indicator values smaller than one ($MS' < 1$) imply a lack of diversity among the obtained set compared with the desired spread, which is most likely due to convergence towards a sub region of the Pareto front. Indicator values larger than one ($MS' > 1$) indicate that the obtained set distributed far away from the Pareto front.

To benchmark the performance of the four algorithms, the scalable function DTLZ2 [9] is invoked. The number of objectives used in this experiment is 3, 4, 6, 8, 10 and 12. The total number of decision variables of the function is $l = M + k - 1$. Where M is the number of objectives and k can be set by user to specify the distance to the Pareto front. According to [9], $k = 10$ is used in DTLZ2.

Table 1: Convergence comparison of the four EMOAs

Obj.	NSGA-II	AR	AR+CD'	Proposed Method
3	0.008513 (0.001236)	0.002630 (0.001219)	0.007697 (0.000969)	0.000590 (0.000481)
4	0.040305 (0.011692)	0.000265 (0.000368)	0.019888 (0.002790)	0.001170 (0.000606)
6	1.533250 (0.139899)	0.000197 (0.000161)	0.061420 (0.010694)	0.001618 (0.000951)
8	2.134350 (0.043475)	0.000314 (0.000427)	0.464933 (0.093250)	0.002187 (0.000788)
10	2.239750 (0.046637)	0.000414 (0.000522)	1.175100 (0.145026)	0.003610 (0.000848)
12	2.266730 (0.050524)	0.000634 (0.000454)	1.489080 (0.120641)	0.004530 (0.000832)

**Figure 5: Distribution of the final solution set by parallel coordinates on four-objective DTLZ2.**

4.2 EXPERIMENTAL SETTINGS

All compared algorithms are given real-valued decision variables. A crossover probability $p_c = 1.0$ and a mutation probability $p_m = 1/l$ (where l is the number of decision variables) are used. The operators for crossover and mutation are simulated binary crossover (SBX) and polynomial mutation with the both distribution indexes are equal to 20 [6]. We run each algorithm independently 100 times. In each run a population of 100 individuals during 300 generations is predefined. For the proposed method, we have set $div = 10$ (i.e., the number of divisions of grid in each dimension) for the test problem with any number of objectives.

The hardware used in compared experiments is a PC with 2.8 GHz Pentium 4 CPU with a memory of 1.00 G, and the operating system is Windows XP.

4.3 EXPERIMENTAL RESULTS

Tables 1 and 2 give the convergence and diversity comparison respectively for all four algorithms over 3, 4, 6, 8, 10 and 12 objectives. The values in the tables correspond to mean and standard deviation. In order to give a visual comparison, Figure 5 and 6 plot

the distribution of the final solution set for four algorithms by parallel coordinates on the problem with 4 and 8 objectives, respectively. Clearly, the proposed method and AR can converge to the Pareto front of the problem with all considered number of objectives. The other two algorithms rapidly decrease in terms of convergence with increasing dimension of objective space, though AR+CD' performs significantly better than NSGA-II.

Note that AR obtains the better convergence values than the proposed method in all case except when the number of objectives is equal to 3. However, from Table 2 and Figure 5 and 6, this result is achieved at the cost of the loss of diversity. In most case, the final solution set obtained by AR locate in a very small part of the Pareto front. For the exception (i.e. the number of objectives is 3), where AR reaches the whole Pareto front, it yet performs worse than the proposed method. Concerning MS', the proposed method is really close to the optimal value and performs considerably best than the other algorithms. Though the final solutions obtained by NSGA-II distribute evenly over the objective space, their position grows away from the Pareto front with the increasing of dimensionality, thereby achieving even worse MS' value.

Table 2: Diversity comparison of the four EMOAs (optimal value is 1)

Obj.	NSGA-II	AR	AR+CD'	Proposed Method
3	1.01734 (0.01231)	0.97032 (0.08933)	0.97929 (0.01359)	1.00091 (0.00473)
4	1.10809 (0.06191)	0.15052 (0.05433)	0.95660 (0.02076)	1.00211 (0.00753)
6	2.98421 (0.11758)	0.10877 (0.03229)	0.95078 (0.02650)	1.00404 (0.01116)
8	3.36105 (0.04029)	0.09047 (0.02903)	1.31038 (0.10297)	1.00294 (0.00754)
10	3.44190 (0.02567)	0.07954 (0.02411)	1.77378 (0.13119)	1.00078 (0.01013)
12	3.46501 (0.01970)	0.06502 (0.02235)	1.87932 (0.10305)	0.99743 (0.01910)

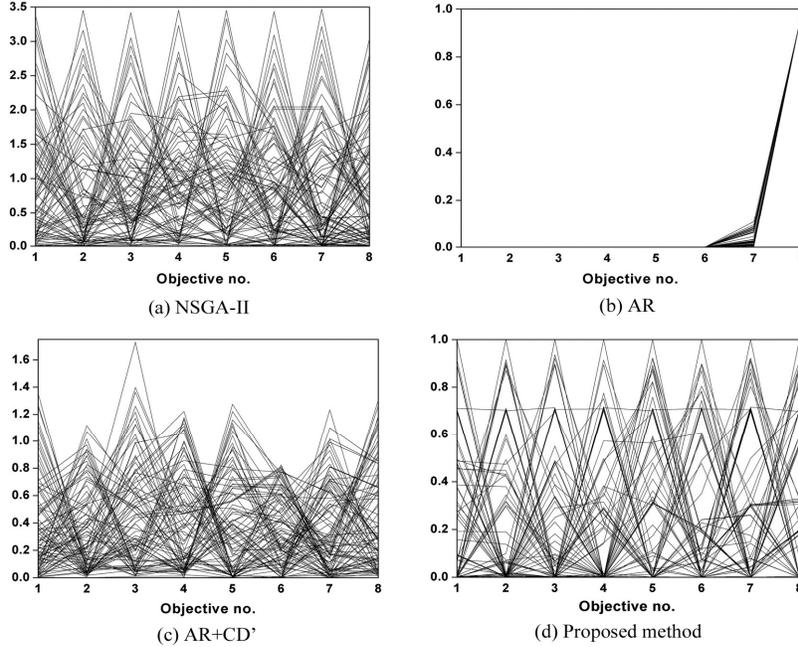


Figure 6: Distribution of the final solution set by parallel coordinates on eight-objective DTLZ2

Further studies with these algorithms have been performed to exhibit the evolutionary trajectories of them. As shown in Figure 7, the proposed method and AR appear to be converging quicker during the whole evolutionary process. Their evolutionary curves are superposed practically. Interestingly, NSGA-II obtains an increasing convergence values compared with its initial state. This occurs probably due to the emphasis of extreme solutions in NSGA-II, which steers individuals towards a misleading search direction. Figure 8 shows the evolutionary trajectories of MS' for four EMOAs. Clearly, the proposed method performs much better than the other algorithms. The solution set of it reaches the whole optimal front at the just about 40 generations. This outstanding result of the proposed method seems to be natural considering the elaborate grid-based fitness strategy that enhances the selection pressure and at the same time avoids a crowded distribution of individuals.

Figure 9 shows the computational costs of the proposed method in comparison with NSGA-II. Clearly, the requirement of both algorithms linearly scales up with the number of objectives. Though the implementation of the proposed method is slower than that of NSGA-II, which is mainly due to its recurrence-mode against batch-mode of NSGA-II in environmental selection, it is perfectly acceptable allowing for the computational cost within 5 seconds even if the number of objectives reaches 12.

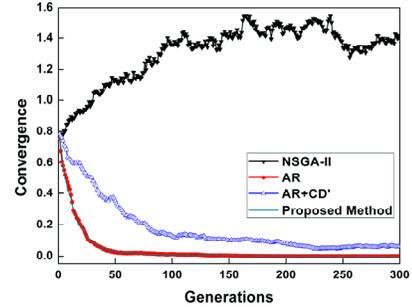


Figure 7: Evolutionary trajectories of convergence metric on six-objective DTLZ2

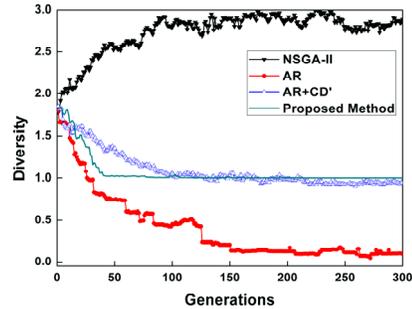


Figure 8: Evolutionary trajectories of diversity metric on six-objective DTLZ2

5. CONCLUSIONS

This paper has presented a strategy that employs the properties of grid to handle many-objective problems. The proposed method constructs an adaptive grid according to evolutionary population and further determines the fitness information of individuals by it. On one hand, three grid-based hierarchical criteria: GR, GCD and GCPD are incorporated into fitness to establish a complete order among individuals. On the other hand, an adaptive fitness adjustment technique in environmental selection is introduced to maintain the diversity of archive set. Simulation experiments have been studied by providing a detailed comparison with other three EMOAs, NSGA-II, AR and AR+CD'. The results reveal that the proposed algorithm has been successful in finding well-converged and well-distributed solution set with only a fraction of computational effort.

One subsequent work is to investigate more test problems for this study, including convex, non-convex, disconnected, linear as well as real-world many objective problems. Moreover, a deeper insight into the behavior of grid-based strategy is also the focus of future work. Especially, the parameter of division in grid for different dimensions would be studied more carefully though the constant setting seems to be performed well in the above study.

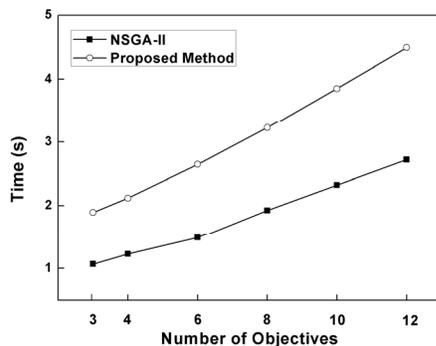


Figure 9: Computational cost on DTLZ2 with different dimensions

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