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The Convergence Analysis of Genetic Algorithm Based on Space Mating

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Abstract

This paper analyzes the convergence properties of the genetic algorithm based on space mating with mutation, crossover and proportional reproduction applied to static optimization on problems. It is proved by means of homogeneous finite Markov chain analysis that genetic algorithm based on space mating will converge to the global optimum. Each process is convergence to the global optimum, at least satisfactory solution under the best individual survives besides the last course. And illuminate a population converge with probability one in the no mutation operator conditions. By comparing the experiment, we can see that the algorithm have better convergence than SGA and consist with the theory.

1. INTRODUCTION

Genetic algorithms (GAs) are general purpose stochastic search methods modeled on natural genetics and the survival of the fittest. It uses the biological evolution, in particular, the terms and principles of genetics to solve problems, has stable biological basis, clear-cut and cognitive science, natural parallel (It uses groups search in place of regular single-point search) and for any type of function (In particular, the type of function can be no expression or expression which can not be accurately calculated) have the highlight characters that can be used and so on. Since this type of technology does Holland [1] put forward, the area of the science of mathematics, computer and information and artificial intelligence have obtain general concern and enthusiastic study.

Although the application of genetic algorithms has been a great success, the theoretical basis (in particular, on the basis of mathematics) remain need to be further study [2, 3]. Some of the main issues of genetic algorithm have not resolved yet. In recent years, the convergence of genetic algorithm research has yielded some progress that provides the basis of theory to the availability of algorithm. Many of studies of different genetic algorithm have been summarized in [4], the value of the number of genetic algorithm has been put forward in [5], and [6] discussed the character of genetic algorithm convergence. But so far, a complete genetic algorithm convergence has few results relatively. Many research put forward variety of improvements of the genetic algorithm, for instance, Hybrid Approach [7], VCGA [8], and HVCSDA [9]. To overcome the problem of premature convergence of the CGA, Zheng etc. proposed the genetic algorithm based on space mating [10]. However, these improvements are based on experiments and lack of theoretical analysis and proof.

This paper proves and illuminates the existence condition of the convergence in the genetic algorithm based on space mating by Markov chain analysis. In section 2 we introduce the algorithm and some definition. Section 3 is devoted to a complexity analysis s and some discussion of the behavior of the algorithm. Section 4 we give the contrast experiment, which is the SGA and the other is SMGA(genetic algorithm based on space mating). A summary of results is given in Section5.

2. GENETIC ALGORITHM BASED ON SPACE MATING

2.1. The Algorithm

Genetic algorithm based on space mating start-up the number of n parallel Process at the beginning. They are used to explore space, so we call the Process is exploration. Once satisfying condition, then select one exploration randomly and take the Process of the model variables out. If empty, then the extracted Pattern added the local Pattern variables directly, otherwise, with the long-distance Pattern mating (space mating). The Process continue to carry out, when the population converge to a solution (May be the local solution), put the "optimal" solution to the development Process, then extract the local Pattern variables, continue the above steps, until received the termination signal so far. To make full use of the explore ability with large mutation probability and the crossover probability, then start-up aided Process. At the beginning, it brings individuals randomly, in the evolution Process, it just exchange the best individual with the development Process. At last, due to the development Process is not convergence, the

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other Process (main Process) deal with it. The algorithm can be sketched as follows:

In the real world, multi-peak value problems are very complex. How to solve this problem? We can introduce Pattern division and space mating. There are some definitions as following.

Definition 1: R is a Pattern vector, and if all of the individuals in a population P can match vector R, we call the population Pattern Space P with R.

Definition 2: R_1 and R_2 are two different vectors. After the crossover operator between R_1 and R_2, there are two new Pattern vectors, namely $R_1^{'}$ and $R_2^{'}$. We will obtain Pattern Space P_1 with $R_1^{'}$ and P_2 with $R_2^{'}$. That is Space Mating.

Definition 3: In evolution population P, at the t generation pi is defined as the optimal individual, whose fitness function f(pi) is maximum, namely f($\max_t$), and F_t is the average of all individuals. $h(p_t) = f(\max_t) - F_t$ is called Distance of population P at the t generation. If t=0, it is initial population. If $h(p_t) < h(p_0)*2/3$, we will consider the population as convergence.

Definition 4: In the evolutional population $P = \{A^1, A^2, ...A^{pop}\}$, one individual $A^i = (a_1^i, a_2^i, ...a_m^i)$, m is the chromosome length and pop is the population size. R is the Pattern vector. We Extract Pattern as following rules. There is a function

$$f_j(\mathrm{a}_j^i) = \begin{cases} 1 & \mathrm{a}_j^i = a_j^1 \\ 0 & \text{else} \end{cases} \quad (1 \le i \le pop;\ j = 1, 2, ...m)$$

And $c_j = \sum_{i=1}^{pop} f_j(a_j^i)$.

(1) If $c_j > pop*2/3$, then the j-th component of R is $a_j^1 (a_j^1 \in \{0,1\})$.

(2) If $c_j < pop*1/3$, then the j-th component of R is $\overline{a}_j^1$.

(3) Else the j-th component of R is *.

Definition 5: Initial population P with Pattern vector R. if R[i] is not *, all of the individuals i -th gene will be R[i]. Else it is the 0 or 1 arbitrarily.

We introduce Process to implement the Space Mating. At the beginning, we initial n Pattern vector R[n], and Initial n populations P[n] with R[n] as the Definition 5. All of the Process run independently, including Sub-Process and Main-Process. Then Sub-Process submits the optimal solution to Main-Process. When the population is convergence (Definition 3), Extract Pattern (Definition 4) and Space Mating (Definition 2). All the Sub-Process will stop at the same until they get the signal from the Main-Process. Figure 1 shows the relation between the Sub-Process and Main-Process. We will describe the algorithm in detail.

Algorithm 1: Sub-Process

Step1: Initial population with Pattern vector R (similar to Definition 1);

Step 2: Implement SGA to generate new population;

Step 3: Submit the optimal solution to Main-Process from the Sub-Process;

Step 4: If the population is convergence (Definition 3) then extract Pattern as Definition 4 and Space Mating with another Sub-Process Pattern vector R' as Definition 2. Then go to **Step 1**;

Step 5: Go to **Step 2**.

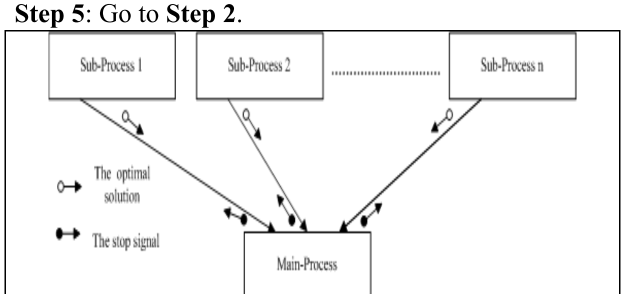

Figure1: The relationship between Sub-Process and Main-Process

Algorithm 1 is only operating framework, and the detailed design has proposed as above. Additionally, we can run the number of n Sub-Process at the same time, and every one has its separate population and Pattern space, although they have the same operators, such as selection, crossover and mutation operators. We submit the best solution to Main-Process population after every generated new population. If there population is convergence, we will call some operators in Pattern space, for example Extract Pattern and Space Mating.

Algorithm 2: Main-Process

Step 1: Initial the population;

Step 2: Update population with the Sub-Process optimal solution;

Step 3: Implement SGA to generate new population. Go to **Step 2**.

Main-Process is also independent with each Sub-Process. At the beginning, we initial the population randomly. But it is different from the SGA, there have two ways to update the population. The one is evolution operator as SGA, and the other is submitted optimal solution from the every Sub-Process.

Algorithm 3: Overall the Space Mating

Step 1: Initial *n* Pattern vector;
Step 2: Run *n* Sub-Process and Main-Process;
Step 3: If stopped condition is not satisfied, go to **Step 2**, else terminate *n* Sub-Process and Main-Process.

In algorithm 3, its framework is similar to the SGA. The stopped condition is the predefinition evolution generation. When the condition is satisfied, all of the Process are terminated and output the results.

2.2. Finite Markov Chains

A future state just related to the current state and nothing to do with the past that is Markov chain. A finite Markov chain describes a probabilistic trajectory over a finite state space S of cardinality |S|=n, where the states may be numbered from 1 to n. The probability $p_{ij}(t)$ of transitioning from state $i \in S$ to state $j \in S$ at step t is called the transition probability from i to j at step t. If the transition probabilities are independent from t, i.e., $p_{ij}(t) = p_{ij}(s)$ for all $i, j \in S$ and for all $s, t \in N$, the Markov chain is said to be homogeneous. For each first arrive, $f_{ii} \in [0,1]$ and $f_{ii} = \sum_{n=1}^{\infty} f_{ii}^{(n)} = 1$ for all $i \in S$. If the finite homogeneous Markov chain can reach the any state of the S^N limitless, it is possible to convergence.

3. MARKOV CHAIN ANALYSIS OF GENETIC ALGORITHM BASED ON SPACE MATING

The genetic algorithm based on space mating can be described as a Markov chain: The state of the algorithm depends only on the genes of the individuals so that the state space i $S = B^N = B^{l \cdot n}$, where n denotes the population size and l is the number of genes. Each element of the state space can be regarded as an integer number in binary representation.

THEOREM 1 The series of population for the genetic algorithm based on space mating is a finite-homogeneous Markov chain.

THEOREM 2 For any states i, make $g_{ij} = f_{ij}$, if j is recurrent, $g_{ij} = 0$, if j is non-recurrent.

PROOF: Let $A_k = e$ (at least have k-n make $X_n(e) = j$)

Clearly

$A_{k+1} \subset A_k$ and $\lim_{k \to \infty} P_i(A_k) = g_{ij}$. (1)

On the other hand

$$P_i(A_{k+1}) = P_i\left\{\bigcup_{m=1}^{\infty}(X_v \neq j, 0 < v < m, X_m = j, X_{m+n} = j)\right\}$$
$$= \sum_{m=1}^{\infty} P_i(X_v \neq j, 0 < v < m, X_m = j) \bullet P_j(X_n(e) = j))$$
$$= \sum_{m=1}^{\infty} f_{ij}^{(m)} P_j(A_k) = f_{ij} P_j(A_k) \quad (2)$$

Because the arbitrary i, repeated iteration (2), note that $P_j(A_1) = f_{jj}$,

$P_i(A_{k+1}) = f_{ij} f_{jj} P_j(A_{k-1}) = \cdots = f_{ij}(f_{jj})^k$,

Let $k \to \infty$,

Thus

$g_{ij} = f_{ij}$, if $f_{jj} = 1$. $g_{ij} = 0$, if $f_{jj} < 1$.

The nature state must be the recurrent and the average time for return i must limited in a finite Markov chain, so it must be the positive recurrent type, and then $g_{ij} = f_{ij} = 1$. Thus from one state to another state with infinite time, in other words, it prove that $\{\vec{X}(n); n \geq 0\}$ can reach any state with infinite time in S^N with probability one. As the explore Process adopt elitist selection[13], then submitted satisfactory solution to the development Process, at the aided Process submitted the better individuals time , the development Process adopt elitist selection too, so the development Process submitted the satisfactory solution to the main Process, the crossover probability is so little and have no mutation operator in main Process, at last, the convergence of the algorithm occur in the main Process, the prove of the convergence of the algorithm can be analyzed as follows:

THEOREM 3 The series of population for the elitist selection is a finite-homogeneous Markov chain

PROOF: First, $\vec{X}(n+1) = T_m T_c T_s\left(\vec{X}(n)\right)$,

Clearly

$\vec{X}(n+1)$ just relate to $\vec{X}(n)$. so $\{\vec{X}(n); n \geq 0\}$ is a finite Markov chain, and the transition probability

$P\left\{\vec{X}(n+1)=\vec{Y}/\vec{X}(n)=\vec{X}\right\}=$

$\prod_{k=1}^{N} P\left\{T\left(\vec{X}(n)\right)_k = Y_k\right\}$, exist $i_0 \in M\left(\vec{X}\right)$ that

Make $Y_N = X_{i_0}$ or else

$P\left\{\vec{X}(n+1)=\vec{Y}/\vec{X}(n)=\vec{X}\right\}=0$.

(1)

Where $i_0 = \arg\max_j \left\{ f(X_j(n)) \right\}$

$M\left(\vec{X}\right)=\left\{i; f\left(X_i\right)=\max\left\{f\left(X_j\right)\right\}\right\}$

So $P\left\{\vec{X}(n+1)=\vec{Y}/\vec{X}(n)\right\}$ have nothing to do with n, then it is a finite-homogeneous Markov chain. The transition probability may recorded as $P\left\{\vec{X},\vec{Y}\right\}=P\left\{\vec{X}(n+1)=\vec{Y}/\vec{X}(n)=\vec{X}\right\}$.

(2)

THEOREM 4 The series of population for the elitist selection converge to the subset M_0^* of the satisfactory set M^* with probability one.

Where $M_0^* = \left\{\vec{Y}=\left(Y_1,\cdots,Y_N\right); Y_N \in M\right\}$

Namely $\lim_{n\to\infty} P\left\{\vec{X}(n)\in M_0^* / \vec{X}(0)=\vec{X}_0\right\}=1$

PROOF: Let X be the best solution of the $f\left(X\right)$.

Because (1) and (2), we know that the property of $P\left\{\vec{X},\vec{Y}\right\}$:

(1) when $\vec{X}$, $\vec{Y}\in M_0^*$, $P\left\{\vec{X},\vec{Y}\right\}>0, P\left\{\vec{Y},\vec{X}\right\}>0$ .Namely, $\vec{X}\leftrightarrow\vec{Y}$

(2) when $\vec{X}\in M_0^*, \vec{Y}\notin M_0^*$, $P\left\{\vec{X},\vec{Y}\right\}=0$ .Namely, $\vec{X} \not\to \vec{Y}$ .And then M_0^* is non-irreducible closed sets of recurrent, $S^N \setminus M_0^*$ is the non-recurrent state set. So

$$\lim_{n\to\infty} P\left\{\vec{X}(n)=\vec{Y}/\vec{X}(0)=\vec{X}_0\right\}=\begin{cases}\pi\left(\vec{Y}\right), \vec{Y}\in M_0^* \\ 0, \vec{Y}\notin M_0^*\end{cases}$$

Clearly

$\lim_{n\to\infty} P\left\{\vec{X}(n)\in M_0^* / \vec{X}(0)=\vec{X}_0\right\}=1$

In the main Process, genetic algorithm based on space mating adopt no mutation operator that make the algorithm converge.

THEOREM 5 If $\left\{X(n); n\geq 0\right\}$ is the series of population for the genetic algorithm based on space mating, $p_m = 0$, H is the homogeneous population of the whole. Namely, $H=\left\{\left(X,\cdots,X\right); X\in S\right\}$

For any $n\geq 1$, $P\left\{X(n)\in H / X(0)\in X\right\}=1$

PROOF: For any $X\in H$, $\varphi(X)=X$, where $\varphi(X)$ is the smallest schema that contains X. Thus when $Y\notin H$, for any $n\geq 1, P\left\{X(n)\in H / X(0)\in X\right\}=1$, and then

$P\left\{X(n)\notin H / X(0)\in H\right\}$

$=\sum_{Y\notin H} P\left\{X(n)=Y / X(0)\in H\right\}$

$=\sum_{Y\notin H} \frac{P\left\{X(n)=Y, X(0)\in H\right\}}{P\left\{X(0)\in H\right\}}$

$=\sum_{Y\notin H}\sum_{X\in H} \frac{P\left\{X(n)=Y / X(0)=X\right\} P\left\{X(0)=X\right\}}{P\left\{X(0)\notin H\right\}}$

=0

Hence,

$P\left\{X(n)\in H / X(0)\in X\right\}=1$

THEOREM 6 If $\left\{X(n); n\geq 0\right\}$ is the series of population for the genetic algorithm based on space mating, p_m=0, H is the homogeneous population, then $\left\{X(n); n\geq 0\right\}$ is converge to H with probability one.

Namely

$\lim_{n\to\infty} P\left\{X(n)\in H\right\}=1$

PROOF: If $X(0)\in H$, **theorem 5** has proven, let $X(0)=\left(X_1,\cdots,X_N\right)\notin H$.

Notes that

$P\left\{X,Y\right\}=P\left\{X(n+1)=Y / X(n)=X\right\}$

$P\left\{X,H\right\}=\sum_{Y\in H} P\left\{X,Y\right\}$

Then

$$P\left\{X,H\right\}\geq \sum_{i=1}^{N} P\left\{X(n+1)=\left(X_i,\cdots,X_i\right) / X(n)=X\right\}$$

$$=\sum_{i=1}^{N}\left[\frac{f\left(X_i\right)^2}{\left(\sum_{j=1}^{N} f\left(X_j\right)\right)^2}\right]^N > 0$$

Note that

$a=\min\left\{P\left(Y,H\right); Y\notin H\right\}$

Then $0 < a < 1$

For $T = \min\{k \ge 1; X(k) \in H\}$ and $k \ge 1$

$$P\{T = k\} = \sum_{Y_1, \cdots, Y_{k-1} \notin H} P\{X, Y_1\} P\{Y_1, Y_2\} \cdots P\{Y_{k-1}, H\}$$

$$\le (1-a)^{k-1}$$

Hence

$$E(T) = \sum_{k=1}^{\infty} k \cdot \{T = k\} \le \sum_{k=1}^{\infty} k(1-a)^{k-1} = \frac{1}{a^2} < \infty$$

Thus $P\{T < \infty\} = 1$,

$$\lim_{k \to \infty} P\{X(k) \in H\} = P\{T < \infty\} = 1$$

From the **theorem 5 and 6**, we can see the main Process is always convergence.

The upper described that the whole algorithm converge to the global optimum in the main Process.

IV. EXPERIMENT

To evaluate the convergence of the proposed algorithm, we compare its result against SGA with elitist. First we introduce 6 test functions as the Figure 2.

From the Figure2, we know that all of test functions are multi-peak value problems apart from (a), and each has the unique optimal solution, for example, test function f's value is -1.803 when $x_1 = x_2 = 2.023$, and others are 0 when $x_1 = x_2 = 0$ or $x = y = 0$.

In this paper, we consider the minimization problem. We design the fitness function based on the test functions. If the individual is closed to the global optimal solution, its fitness value approaches 1. Figure 3 shows the mean and average fitness value of every 10 generations, which are obtained by the Space Mating genetic algorithm (SMGA) and Simple genetic algorithm with elitist (SGA).

(a): Sphere Model: $f(x,y) = x^2 + y^2$ $(x, y \in [-10,10])$	(b): Rastrigrin: $f(x,y) = 20 + x^2 - 10\cos(2\pi x) + y^2 - 10\cos(2\pi y)$ $(x, y \in [-10,10])$	(c): Schaffer's f6: $f(x,y) = 0.5 + \frac{\sin^2\sqrt{x^2+y^2} - 0.5}{(1+0.001\times(x^2+y^2)^2)^2}$ $(x, y \in [-10,10])$
(d):Griewangk's: $f(x) = \sum_{i=1}^{N} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos(\frac{x_i}{\sqrt{i}}) + 1$ $(x_1, x_2 \in [-10,10])$	(e)Ackley's Path function: $f(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(-\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x)) + 20 + e$ $(x_1, x_2 \in [-10,10])$	(f)Michalewicz's: $f(x) = -\sum_{i=1}^{n} \sin(x_i)\sin^{20}(\frac{i \times x_i^2}{\pi})$ $(x_1, x_2 \in [0, \pi])$

Figure2 : Function formula

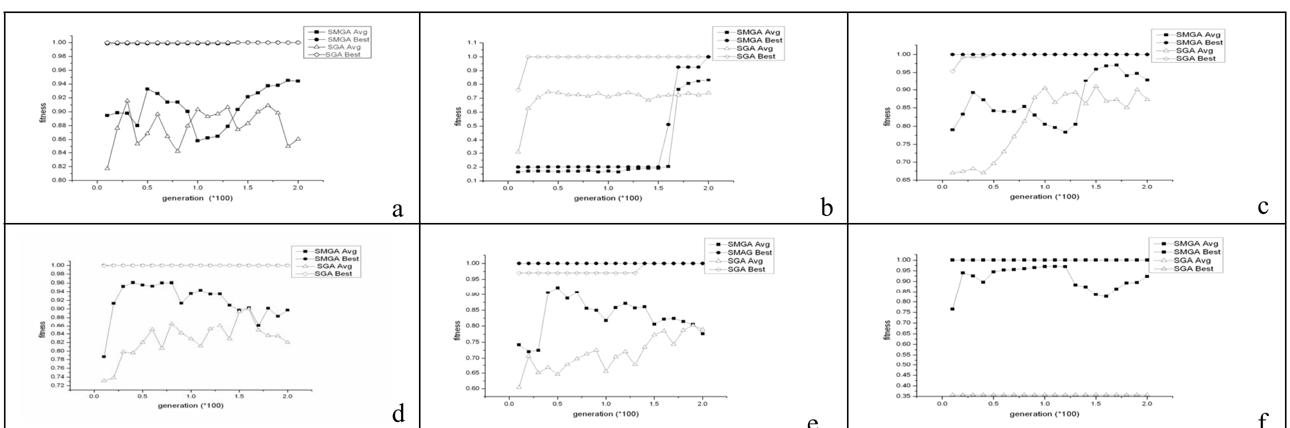

Figure 3: The mean and average fitness value every 10 generations

From (a), (b), (c) in the Figure 3, we find that two algorithm can find the global optimal solution. The best fitness curve achieves 1.0, that's to say, there have a individual in the population is the global solution. The other two curves interpret us whether the population is convergence or not. If the average value curve is closed to 1.0, the population approaches convergence. The three figures tell us that the SMGA doesn't dominate the SGA completely. But the former is better than the latter at many local populations, for example, between 40 and 90 generation and after 140 generation and so on the former is better than the latter in Figure (a).

As the (d) and (e), two methods could find the best solutions. SMGA outperform the SGA obviously, because the SMGA 's average fitness value curves are close to 1.0 than the SGA's.

The last figure (f) shows SMGA could find the best individual, however, the SGA can't. SGA's average and best curves are about the same one, and it is far from 1.0. In other words, SGA could only obtain the local solution. At the same time, SMGA should be convergence with the lapse of generation.

All the experiments show the SMGA exceed the SGA, not only the convergence but also finding the global optimal solution. The results of experiment are concordant with the theory.

4. CONCLUSION

In this paper, the model of genetic algorithm based on space mating have been set-up, the algorithm divided the whole search space into non-intersect sub-space through the space mating, but also adopt the way of the Process to make the sub-space as the operation object, It has been optimized and reduce continuously by itself.. Algorithm has always maintained a low probability of crossover. Therefore, it is not diversity of groups suddenly at some time, but to maintain diversity of groups on the whole, at the same time, continue to accumulate useful information, the algorithm converge to the global optimum at the end and overcome the problem of premature converge effectively .In this paper, the theory of genetic algorithm based on space mating has been analyzed, and discussed

The convergence of algorithm and the problem of objects under no mutate operator conditions, Theoretical Analysis of Algorithms will provide the necessary theoretical basis, and indicate the direction of improving the algorithm. Through the contrast of the six experiments, clearly, the algorithm in the theory of convergence, convergence rate, the complexity of the time and so on have better result.

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